Current Issues on 3D Volumetric Positioning Accuracy: Measurement, Compensation and Definition

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ABSTRACT

Traditionally, manufacturers have ensured part accuracy by linear calibration of each machine tool axis. The conventional definition of the 3-D volumetric positioning error is the root mean square of the three-axis displacement error. 20 years ago, the dominate error is the lead screw pitch error of 3 axes. This definition is adequate. However, now the machine accuracy has been improved with better lead screw, linear encoder and compensation, the dominate errors become the squareness errors and straightness errors. Hence the above definition is inadequate.

During the past years, the industry has seen demand emerge for the "volumetric accuracy" specification on machine tools. One hurdle remains: a standard definition so that everyone measures volumetric accuracy with the same yardstick. The issue has been discussed in many Standards Committees, machine tool builders and the metrology community. Reported here are, a new 3D volumetric positioning error measurement and compensation technique, proposed definitions or measures of 3 D volumetric positioning errors of a CNC machine tool, and its verification.

Keywords: Machine Accuracy, Positioning errors, Error compensation, 3D volumetric errors.

1. INTRODUCTION

World competition requires good quality or accurate parts. Hence the CNC machine tool positioning accuracy becomes very important. Twenty years ago, the largest machine tool positioning errors are lead screw pitch error and thermal expansion error. Now, most of the above errors have been reduced by better lead screw, linear encoder and pitch error compensation. The largest machine tool positioning errors become squareness errors and straightness errors. Hence, to achieve higher 3D volumetric positioning accuracy, all 3 displacement errors, 6 straightness errors and 3 squareness errors have to be measured. Using a conventional laser interferometer to measure these errors is rather difficult and costly. It usually takes days of machine down time and experienced operator to perform these measurements.

It has been proposed to use the body diagonal displacement errors to define the volumetric positioning error [1]. However, the relations between the measured body diagonal displacement errors and the 21 rigid body errors were not clear at that time. Hence the current issues in machine positioning errors are how to measure these errors with ease, how to compensate these errors and how to define and to determine the 3D volumetric positioning error of CNC machine tools. The definition should be directly linked to the 3D positioning errors and also practical to measure or determine such that it will be accepted by machine tool builders and users.

Reported here are, a new 3D volumetric positioning error measurement and compensation technique, proposed definitions or measures of 3 D volumetric positioning errors of a CNC machine tool, and its verification.

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2. RIGID BODY POSITIONING ERRORS

The positioning error in an arbitrary point within the machine working volume is composed by the positioning errors of the individual axes. In a rigid body motion there are six different geometric errors of a single axis, namely, the linear displacement error, the straightness errors - horizontal and vertical and the angular errors - pitch, yaw and roll. Hence for a three-axis machine tool there are 18 errors plus the three squareness errors in the individual machine coordinate planes. Therefore there are a total of 21 errors. These 21 rigid body errors [2] can be expressed as the followings.

Linear displacement errors: \( D_x(x), D_y(y), \) and \( D_z(z) \)
Vertical straightness errors: \( D_y(x), D_x(y), \) and \( D_x(z) \)
Horizontal straightness errors: \( D_z(x), D_z(y), \) and \( D_y(z) \)
Roll angular errors: \( A_x(x), A_y(y), \) and \( A_z(z) \)
Pitch angular errors: \( A_y(x), A_x(y), \) and \( A_x(z) \)
Yaw angular errors: \( A_z(x), A_z(y), \) and \( A_y(z) \)
Squareness errors: \( \Theta_{xy}, \Theta_{yz}, \Theta_{zx}, \)
where, \( D \) is the linear error, subscript is the error direction and the position coordinate is inside the parenthesis, \( A \) is the angular error, subscript is the axis of rotation and the position coordinate is inside the parenthesis.

In most cases, coordinate measuring machines (CMM) and machine tools can be classified into four configurations [3]. They are the FXYZ, XFYZ, XYFZ, and XYZF. Here, the axis before F show available motion directions of the work piece with respect to the base, and the letters after F show the available motion directions of the tool (or probe) with respect to the base. For example, in FXYZ the work piece is fixed, and in XYZF the tool is fixed.

Using the stacking model, namely, the displacement errors caused by the pitch, yaw and roll angular errors are the Abbe offset times the angular errors and the signs are determined by the right-hand-rule, the positioning errors in the \( x, y, \) and \( z \) directions were derived in the followings [4,5].

For the configuration FXYZ, \( x \)-axis is mounted on a fixed base, \( y \)-axis is mounted on the \( x \)-axis and \( z \)-axis is mounted on the \( y \)-axis. Hence for \( x \)-axis movement, there is no Abbe offset on \( x \) and the angular error terms are \( y^*A_x(x), y^*A_z(x), \) and \( z^*A_y(x); \) for \( y \)-axis movement, there are no Abbe offset on \( x \) and \( y \) and the angular error terms are \( -z^*A_x(y) \) and \( z^*A_y(y) \); for \( z \)-axis movement, there are no Abbe offsets on \( x, y \) and \( z \) and there is no angular error term. The results are the same as derived by positioning vector and rotation matrices in Ref [4].

\[
\begin{align*}
E_x(x, y, z) &= [D_x(x) - y^*A_z(x) + z^*A_y(x)] + [D_y(y) + z^*A_y(y)] + [D_z(z)] \quad (1) \\
E_y(x, y, z) &= [D_y(y) - z^*A_x(x)] + [D_y(y) - z^*A_x(y)] + [D_y(y)] \quad (2) \\
E_z(x, y, z) &= [D_z(z) + y^*A_x(x)] + [D_y(y)] + [D_z(z)]. \quad (3)
\end{align*}
\]

Similarly for the configuration XFYZ, \( x \)-axis is mounted on a fixed base, \( y \)-axis is mounted on the \( x \)-axis and \( z \)-axis is mounted on the \( y \)-axis. Hence for \( x \)-axis movement, there are all 3 Abbe offsets and the angular error terms are \( -x^*A_y(x), x^*A_z(x), -y^*A_z(x), -z^*A_x(x), \) and \( z^*A_y(x); \) for \( y \)-axis movement, there are no Abbe offsets on \( x \) and \( y \) and the angular error terms are \( -z^*A_x(y) \) and \( z^*A_y(y) \); for \( z \)-axis movement, there are no Abbe offsets on \( x, y \) and \( z \) and there is no angular error term. The results are the same as derived in Ref [4].

\[
\begin{align*}
E_x(x, y, z) &= [D_x(x) - y^*A_z(z) + z^*A_y(x)] + [D_y(y) + z^*A(y)] + [D_z(z)] \quad (4) \\
E_y(x, y, z) &= [D_y(y) + x^*A(x) - z^*A_x(x)] + [D_y(y) - z^*A_x(y)] + [D_z(z)] \quad (5) \\
E_z(x, y, z) &= [D_z(z) - x^*A_y(x) + y^*A_x(x)] + [D_y(y)] + [D_z(z)]. \quad (6)
\end{align*}
\]

Similarly for the configuration XYFZ, \( x \)-axis is mounted on a fixed base, \( y \)-axis is mounted on the \( x \)-axis and \( z \)-axis is mounted on a fixed base. Hence for \( x \)-axis movement, there is no Abbe offset on \( y \) and the angular error terms are \( -x^*A_y(x), x^*A(z(x), -z^*A(x), \) and \( z^*A_y(x); \) for \( y \)-axis movement, there are all 3 Abbe offsets, and the angular terms are \( x^*A_y(y), x^*A(z(x), -y^*A(z(x), -z^*A(x) and z^*A_y(x); \) for \( z \)-axis movement, there is no Abbe offset on \( x, y \) and \( z \) and no angular term. The results are the same as derived in Ref [4].
Finally for the configuration XYZF, x-axis is mounted on a fixed base, y-axis is mounted on the x-axis and z-axis is mounted on the y-axis and the spindle is fixed. Hence for x-axis movement, there are no Abbe offsets on x and y and the angular error terms are -z*Ax(x), and z*Ay(x); for y-axis movement, there is no Abbe offset on z, and the angular error terms are -x*Ay(y), x*Az(y), y*Ax(y), -y*Az(y); for z-axis movement, there are all 3 Abbe offsets and the angular error terms are -x*Ay(z), x*Az(z), y*Ax(z), -y*Az(z), -z*Ax(z) and z*Ay(z). The results are the same as derived in Ref [4].

\[
\begin{align*}
Ex(x, y, z) &= [Dx(x) + z*Ay(x)] + [Dx(y) - y*Az(y) + z*Ay(y)] + [Dx(z)] \\
Ey(x, y, z) &= [Dy(x) + x*Az(x) - z*Ax(x)] + [Dy(y) + x*Az(y) - z*Ax(y)] + [Dy(z)] \\
Ez(x, y, z) &= [Dz(x) - x*Ay(x)] + [Dz(y) - x*Ay(y) + y*Ax(y)] + [Dz(z)].
\end{align*}
\]  

To compensate these positioning errors, most machine controllers can perform compensation for repeatable lead screw or encoder errors on each axis of motion. Usually this is called pitch error compensation. Some machines with advanced controllers can provide compensation for repeatable displacement errors, vertical and horizontal straightness errors and squareness errors. Most machine controllers do not have the capability of compensate angular error and usually the angular error times the Abbe offset is included in the measured straightness errors. However, many times the tool offsets are different, the effect of angular errors are different. In many advanced controllers, the non-rigid body repeatable errors can be compensated by a three dimensional grid point error map. In such 3-D error compensation, the error compensation for an arbitrary interior point is interpolated by the surrounding 8 error compensation grid points. The error values of these 8 grid points are measured and input to the control.

### 3. BODY DIAGONAL DISPLACEMENT MEASUREMENT

Using a conventional laser interferometer to measure the straightness and squareness errors is rather difficult and costly. It usually takes days of machine down time and experienced operator to perform these measurements. For those reasons the body diagonal displacement error defined in the ASME B5.54 or ISO 230-6 standard is a good quick check of the volumetric error [6,7]. Furthermore, it has been used by Boeing Aircraft Company and many others for many years with very good results and success.

Briefly, similar to a laser linear displacement measurement, instead of pointing the laser beam in the axis direction, pointing the laser beam in the body diagonal direction. Mount a retroreflector on the spindle and move the spindle in the body diagonal direction. Starting from the zero position and at each increment of the three axes, which are moved together to reach the new position along the diagonal, the displacement error is measured. There are 4 body diagonal directions as shown in Fig. 1. The accuracy of each position along the diagonal depends on the positioning accuracy of the three axes, including the straightness errors, angular errors and squareness errors. Hence the 4 body diagonal displacement measurements are a good method for the machine verification.

The relations between the measured 4 body diagonal displacement errors and the 21 rigid body errors have been derived in Ref [4].

For the FXYZ, the measured error DR at each increment can be expressed as [4,5],

\[
\begin{align*}
\text{DRppp} &= a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) \\
&+ a/r*[Dx(y) + y*Oxy] + b/r*[Dy(y) + x*Oxy] + c/r*Dz(y) \\
&+ a/r*[Dx(z) + z*Oz] + b/r*[Dy(z) + z*Oz] + c/r*Dz(z) \\
&+ Ay(x)*ac/r - Az(x)*ab/r + Ay(y)*ac/r - Ax(y)*bc/r. \\
\text{DRnpp} &= -a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) \\
&+ a/r*[Dx(y) + y*Oxy] + b/r*[Dy(y) + x*Oxy] + c/r*Dz(y)
\end{align*}
\]
\[ \begin{align*}
- a/r \times [Dx(z) + z \cdot \delta z] + b/r \times [Dy(z) + z \cdot \delta yz] + c/r \times Dz(z) \quad (14) \\
- Ay(x) \times ac/r + Az(x) \times ab/r - Ay(y) \times ac/r - Ax(y) \times bc/r.
\end{align*} \]

\[ \begin{align*}
DR_{pnp} = a/r \times Dx(x) - b/r \times Dy(x) + c/r \times Dz(x) \\
+ a/r \times [Dx(y) + y \cdot \delta xy] - b/r \times Dy(y) + c/r \times Dz(y) \\
+ a/r \times [Dx(z) + z \cdot \delta zx] - b/r \times [Dy(z) + z \cdot \delta yz] + c/r \times Dz(z) \\
+ Ay(x) \times ac/r + Az(x) \times ab/r - Ay(y) \times ac/r + Ax(y) \times bc/r. \quad (15)
\end{align*} \]

\[ \begin{align*}
DR_{ppn} = a/r \times Dx(x) + b/r \times Dy(x) - c/r \times Dz(x) \\
+ a/r \times [Dx(y) + y \cdot \delta xy] + b/r \times Dy(y) - c/r \times Dz(y) \\
+ a/r \times [Dx(z) + z \cdot \delta zx] + b/r \times [Dy(z) + z \cdot \delta yz] - c/r \times Dz(z) \\
- Ay(x) \times ac/r - Az(x) \times ab/r - Ay(y) \times ac/r + Ax(y) \times bc/r. \quad (16)
\end{align*} \]

where the subscript ppp means body diagonal with all x, y and z positive; npp means body diagonal with x negative, y and z positive; pnp means body diagonal with y negative, x and z positive; and ppn means body diagonal with z negative, x and y positive. Also a, b, c and r are increments in x, y, z, and body diagonal directions respectively. The body diagonal distance can be expressed as \( r^2 = a^2 + b^2 + c^2 \).

In the FXYZ configuration, there are 4 angular error terms, \( Ay(x) \times ac/r \), \( Az(x) \times ab/r \), \( Ay(y) \times ac/r \) and \( -Ax(y) \times bc/r \). In the XFYZ configuration most of the angular error terms are cancelled and only 2 angular error terms, \( Ay(y) \times ac/r \) and \( -Ax(y) \times bc/r \) are left. Similarly in the XYFZ configuration only 2 angular error terms, \( Az(x) \times ab/r \) and \( -Ax(x) \times bc/r \) are left. Finally in the XYZF configuration there are 4 angular error terms, \( Ay(x) \times ac/r \), \( -Ax(x) \times ab/r \), \( Ay(y) \times ac/r \) and \( -Ax(y) \times bc/r \) exactly the same as in the FXYZ configuration. Since the configuration for most common horizontal machining centers and vertical machining centers are XFYZ and XYFZ respectively, we can conclude that the body diagonal displacement measurement are not sensitive to angular errors.

The 4 body diagonal displacement errors are sensitive to all of the 9 linear errors and some angular errors. Hence it is a good measurement of the 3D volumetric positioning errors. The errors in the above equations may be positive or negative and they may cancel each other. However, the errors are statistical in nature, the probability that all of the errors will be cancelled in all of the positions and in all of the 4 body diagonals are theoretically possible but very unlikely. Hence it is indeed a good measurement of volumetric positioning accuracy.

4. VECTOR OR SEQUENTIAL STEP DIAGONAL DISPLACEMENT MEASUREMENT

To overcome the limitations in the 4 body diagonal displacement measurement, a sequential step diagonal or vector technique [8,9,10] has been developed by Optodyne. The basic concept of the vector method is that the laser beam direction is not parallel to the motion direction. Hence, the measured displacement errors are sensitive to the errors both parallel and perpendicular to the direction of the motion. More precisely, the measured linear errors are the vector sum of errors, namely, the displacement errors (parallel to the linear axis), the vertical straightness errors (perpendicular to the linear axis), and horizontal straightness errors (perpendicular to the linear axis and the vertical straightness error direction), projected to the direction of the laser beam. Furthermore, collect data with the laser beam pointing in 4 different diagonal directions; all 9 error components can be determined. Since the errors of each axis of motion are the vector sum of the 3 perpendicular error components, we call this measurement a “vector” method.

For conventional body diagonal displacement measurement all 3 axes move simultaneously, the displacement is a straight line along the body diagonal; hence a laser interferometer can be used to do the measurement. However, for the vector measurement described here, the displacements are along the x-axis, then along the y-axis and then along the z-axis. The trajectory of the target is not parallel to the diagonal direction as shown in Fig. 2. The deviations from the body diagonal are proportional to the size of the increment X, Y, or Z. A conventional laser interferometer will be way out of alignment even with an increment of a few mm.

To tolerate such large lateral deviations, a Laser Doppler Displacement Meter (LDDM™) using a single
aperture laser head and a flat-mirror as the target can be used [8,9]. A schematic showing the flat mirror positions during the measurement steps is shown in Fig. 3. Here the flat-mirror target is mounted on the machine spindle and it is perpendicular to the laser beam direction.

In summary, in a conventional body diagonal measurement all 3 axes move simultaneously along a body diagonal and collect data at each preset increment. In the vector measurement all 3 axes move in sequence along a body diagonal and collect data after each axis is moved. Hence, not only 3 times more data are collected, the error due to the movement of each axis can also be separated. Since each body diagonal measurement collected 3 sets of data, there are 12 sets of data. Hence, there are enough data to solve the 3 displacement errors, 6 straightness errors and the 3 squareness errors.

The setup is simple and easy and the measurement can be performed in a few hours instead of a few days using a conventional laser interferometer. Hence it is more practical. The measured volumetric positioning errors can also be used to generate a 3D volumetric compensation table to correct the positioning errors and achieve higher positioning accuracy. For example, the Siemens 840 controllers has 18 tables for linear and sag compensations. For each axis, there are displacement error forward, displacement error backward, horizontal straightness forward, horizontal straightness backward, vertical straightness forward and horizontal straightness backward, a total of 6 tables for each axis and a total of 18 tables for 3 axes.

For the Fanuc 30i/31i/32i controller with 128-point option has 9 compensation tables. The first 3 tables are for displacement errors (or pitch errors), for x-axis (Dx(x)), y-axis (Dy(y)) and z-axis (Dz(z)). The next 6 tables are for straightness errors, moving axis = X, compensation axes = Y (Dy(x)) and Z (Dz(x)); moving axis = Y, compensation axes = X (Dx(y)) and Z (Dz(y)); and moving axis = Z, compensation axes = X (Dx(z)) and Y (Dy(z)). For all 9 tables, the unit, the comp unit, the comp algorithm, comp digits and travel direction should all be the same. The increment and the reference should be the same for x-axis, y-axis and z-axis.

5. PROPOSED DEFINITIONS OF 3D VOLUMETRIC ERROR & VERIFICATION

Volumetric accuracy for movements in X,Y,Z V(XYZ) is the maximum range of relative deviations between actual and ideal position in X, Y, Z and orientations in A, B, C for X, Y, Z movement in the volume concerned, where the deviations are relative deviations between the tool side and the work piece side of the machine tool.

Assuming rigid body motion, the formulae for the 6 errors in the directions of X, Y, and Z, and rotary axes A, B, and C as the followings [7],

\[
V(XYZ, X) = \text{deviations in X at } XYZ, \\
= Dx(x) + Dx(y) + Dx(z), \quad (17)
\]

\[
V(XYZ, Y) = \text{deviations in Y at } XYZ, \\
= Dy(x) + Dy(y) + Dy(z), \quad (18)
\]

\[
V(XYZ, Z) = \text{deviations in Z at } XYZ, \\
= Dz(x) + Dz(y) + Dz(z), \quad (19)
\]

\[
V(XYZ, A) = \text{angular deviations around X at } XYZ, \\
= Ax(x) + Ax(y) + Ax(z), \quad (20)
\]

\[
V(XYZ, B) = \text{angular deviations around Y at } XYZ, \\
= Ay(x) + Ay(y) + Ay(z), \quad (21)
\]

\[
V(XYZ, C) = \text{angular deviations around Z at } XYZ, \\
= Az(x) + Az(y) + Az(z). \quad (22)
\]

Here the squareness errors are included in the straightness errors. The angular errors are small and can be treated as scalar.

**Definition 1:**
The amplitude of the volumetric error can be defined as the RMS of the 3 linear deviations and the
amplitude of the volumetric angular error can be defined as the RMS of the 3 angular deviations.

\[
V(XYZ, R) = \sqrt{V(XYZ, X) * V(XYZ, X) + V(XYZ, Y) * V(XYZ, Y) + V(XYZ, Z) * V(XYZ, Z)}, \quad (23)
\]

\[
V(XYZ, W) = \sqrt{V(XYZ, A) * V(XYZ, A) + V(XYZ, B) * V(XYZ, B) + V(XYZ, C) * V(XYZ, C)}. \quad (24)
\]

The volumetric accuracy and volumetric angular accuracy can be defined as the maximum range over the working space.

\[
R_{\text{max}} = \text{Max}\{V(XYZ, R)\} \quad (25)
\]

\[
W_{\text{max}} = \text{Max}\{V(XYZ, W)\}. \quad (26)
\]

**Definition II:**

The maximum range of error in each direction can be expressed as,

\[
X_{\text{max}} = \text{Max}\{V(XYZ,X)\} - \text{min}\{V(XYZ, X)\}, \quad (27)
\]

\[
Y_{\text{max}} = \text{Max}\{V(XYZ,Y)\} - \text{min}\{V(XYZ, Y)\}, \quad (28)
\]

\[
Z_{\text{max}} = \text{Max}\{V(XYZ,Z)\} - \text{min}\{V(XYZ, Z)\}, \quad (29)
\]

\[
A_{\text{max}} = \text{Max}\{V(XYZ,A)\} - \text{min}\{V(XYZ, A)\}, \quad (30)
\]

\[
B_{\text{max}} = \text{Max}\{V(XYZ,B)\} - \text{min}\{V(XYZ, B)\}, \quad (31)
\]

\[
C_{\text{max}} = \text{Max}\{V(XYZ,C)\} - \text{min}\{V(XYZ, C)\}. \quad (32)
\]

The volumetric accuracy and volumetric angular accuracy can be defined as the RMS of the maximum range of error in each direction.

\[
R'_{\text{max}} = \sqrt{X_{\text{max}} \cdot X_{\text{max}} + Y_{\text{max}} \cdot Y_{\text{max}} + Z_{\text{max}} \cdot Z_{\text{max}}}, \quad (33)
\]

\[
W'_{\text{max}} = \sqrt{A_{\text{max}} \cdot A_{\text{max}} + B_{\text{max}} \cdot B_{\text{max}} + C_{\text{max}} \cdot C_{\text{max}}}. \quad (34)
\]

Both of these 2 definitions are valid definitions. However, to determine the volumetric error, it requires extensive and time consuming measurement. A third definition is to use the 4 body diagonal displacement errors to define the volumetric accuracy.

**A proposed new definition of 3D volumetric error based on the body diagonal errors**

The performance or accuracy of a machine tool is determined by 3-D volumetric positioning error, which includes linear displacement error, straightness error, angular error, and thermally induced error. The body diagonal displacement error defined in ASME B5.54 or ISO 230-6 is a good quick check of volumetric error. All the errors will contribute to the four-body diagonal displacement errors. The B5.54 tests have been used by Boeing Aircraft Co. and others for years.

When using body diagonal displacement error measurement, body diagonal error (Ed) does not include squareness errors. But Ed is currently defined in ISO 230-6 and ASME B5.54 as a measure of volumetric error. Squareness errors can be included, and our new proposed measure volumetric error, ESd, is defined below,

\[
ESd = \text{Max}[Dr(r)ppp/nnn, Dr(r)npp/nnn, Dr(r)pnp/nnn, Dr(r)ppn/nnn] - \text{min}[Dr(r)pp/nnn, Dr(r)npp/nnn, Dr(r)pnp/nnn, Dr(r)ppn/nnn].
\]

To demonstrate this new definition, measurements were performed on 10 selected CNC machine tools, representing the modern mid-size CNC machining centers [1]. Eight were made by the German manufacturer Deckel Maho Gildemeister (DMG), one by the UK Bridgeport and one by the Czech company Kovosvit MAS. The DMG machines are for better illustration inscribed with a number behind each type description (e.g. DMU80T-2). A brief description of the 10 machines is in Table 1.

The measurement results are shown in Table 2. Measurements according to ISO 230-2 were performed along the three edges of the machine working volume. These are identified by the marks I, II and III. The
angular errors are derived from the linear positioning by respecting the Abbe offsets. The diagonal positioning accuracy is described by the parameter \( Ed \) (diagonal systematic deviation of positioning) according to ISO 230-6. The remaining geometric errors were evaluated from the laser vector method.

As compared with \( ELSv \), the actual volumetric positioning errors, the \( ESd \) is underestimated the 3D volumetric position error but relatively stable and not affected by the squareness errors. For more quantitative comparison, a multiple factor, \( M \) is defined as the ratio of \( ELSv/ESd \). Hence the true 3D volumetric error \( ELSv \) can be obtained by multiple the \( ESd \) by \( M \). The measured \( M \) varies from 1.4 to 1.9 which is relatively constant as compared with other definitions. Hence the \( ESd \) is a good estimate of 3D volumetric position error.

6. Summary and conclusion

Three definitions of the 3D volumetric positioning error have been provided. The positioning errors of 10 CNC machine tools have been measured. Based on these measurement results, the 3D volumetric errors using various definitions can be calculated. It is concluded that the laser body diagonal displacement measurement in the ASME B5.54 or ISO 230-6 machine tool performance measurement standards is a quick check of the volumetric positioning error and the value \( ESd \) is a good measure of the volumetric error.

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References

Table 1, Machine parameters of the 10 selected modern machining centers

<table>
<thead>
<tr>
<th>Machine No.</th>
<th>Machine id.</th>
<th>Manufacturer</th>
<th>Type</th>
<th>Axis stroke (X/Y/Z) mm</th>
<th>Control sys.</th>
<th>Service hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DMC60H-1</td>
<td>DMG</td>
<td>horizontal</td>
<td>600 / 560 / 560</td>
<td>Sinumerik</td>
<td>2589</td>
</tr>
<tr>
<td>2</td>
<td>DMC60H-2</td>
<td>DMG</td>
<td>horizontal</td>
<td>600 / 560 / 560</td>
<td>Sinumerik</td>
<td>1655</td>
</tr>
<tr>
<td>3</td>
<td>DMC65V-1</td>
<td>DMG</td>
<td>vertical</td>
<td>650 / 500 / 500</td>
<td>Sinumerik</td>
<td>3550</td>
</tr>
<tr>
<td>4</td>
<td>DMC65V-2</td>
<td>DMG</td>
<td>Vertical</td>
<td>650 / 500 / 500</td>
<td>Sinumerik</td>
<td>3338</td>
</tr>
<tr>
<td>5</td>
<td>DMU80T-1</td>
<td>DMG</td>
<td>Vertical</td>
<td>880 / 630 / 630</td>
<td>Heidenhein iTNC530</td>
<td>2847</td>
</tr>
<tr>
<td>6</td>
<td>DMU80T-2</td>
<td>DMG</td>
<td>vertical</td>
<td>880 / 630 / 630</td>
<td>Heidenhein iTNC430</td>
<td>4081</td>
</tr>
<tr>
<td>7</td>
<td>DMU80T-3</td>
<td>DMG</td>
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Table 2: Measurement results

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Fig. 1, Shows the 4 body diagonal directions

Fig. 2, Vector measurement trajectory, the laser is pointing in the ppp diagonal Direction. Move Dx, stop, collect data, move Dy, stop, collect data and, move Dz, stop, collect data and so on.
Fig 3, Shows the sequential step diagonal or vector technique

Fig. 4, A photo of actual laser setup for the vector measurement